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Existence and global attractivity of positive periodic solutions for a Holling II two-prey one-predator system

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Abstract

In this paper, a Holling II two-pery one-predator system is investigated. Based on the continuation theorem of coincidence degree theory and by constructing a suitable Lyapunov function, we derive a set of sufficient conditions that guarantee the existence of at least a positive periodic solution and global attractivity of periodic solutions.

Mathematics Subject Classification 2000: 34K20; 34C25

Keywords: Predator-prey system, periodic solution, Holling II functional response, coincidence degree theory, global attractivity

Introduction

In population dynamics, the functional response, which is a key element in all predator-prey interaction, is referred to the number of prey eaten per predator per unit time as a function of prey density. Based on a lot of experiments, Holling [1] suggested the following three different kinds of functional response for different species to model the phenomenon of predation:

$$(1)\, p_1(x) = ax,\ (2)\, p_2(x) = \frac{ax}{m+x},\ (3)\, p_3(x) = \frac{ax^2}{m+x^2},$$

where $x(t)$ represents the prey density at time t. Functions $p_1(x)(i = 1, 2, 3)$ are referred to the Holling type I, II, and III functional response, respectively. $a > 0$ denotes the search rate of the predator, $m > 0$ is the half-saturation constant. Predator-prey systems with Holling type functional response have been investigated extensively, for example, Liu and Chen [2] made a discussion on complex dynamics of Holling type II Lotka-Volterra predator-prey model with impulsive perturbations on the predator. Song and Li [3] studied the linear stability of trivial periodic solution and semi-trivial periodic solutions and the permanence of the periodic predator-prey model with modified Leslie-Gower Holling-type II schemes and impulsive effect. Liu and Xu [4] investigated the existence of periodic solution for a delay one-predator and two-prey system with Holling type-II functional response. Agiza et al. [5] considered the chaotic phenomena of a discrete prey-predator model with Holling type II. Pei et al. [6] analyzed the extinction and permanence for one-prey multi-predators of Holling type II function response system with impulsive biological control. For more knowledge about this theme, one can see [7-18].

In 2007, Song and Li [19] had considered the dynamical behaviors of the following Holling II two-prey one predator system with impulsive effect

$$\begin{cases} \left.\begin{array}{l} \dot{x}_1(t) = x_1(t)\left[b_1 - x_1(t) - \alpha x_2(t) - \dfrac{\eta z(t)}{1+\omega_1 x_1(t),}\right], \\ \dot{x}_2(t) = x_2(t)\left[b_2 - \beta x_1(t) - x_2(t) - \dfrac{\eta z(t)}{1+\omega_2 x_2(t)}\right], \\ \dot{z}(t) = z(t)\left[-b_3 + \dfrac{d\eta x_1(t)}{1+\omega_1 x_1(t)} + \dfrac{d\eta x_2(t)}{1+\omega_2 x_2(t)}\right], \end{array}\right\} t \neq nT, \\ \left.\begin{array}{l} \Delta x_1(t) = -p_1 x_1(t), \\ \Delta x_1(t) = -p_2 x_2(t), \\ \Delta z(t) = 0, \end{array}\right\} t = nT, \end{cases} \tag{1}$$

where $x_i(t)(i = 1, 2)$ is the population size of prey (pest) species and $z(t)$ is the population size of predator (natural enemies) species, b_i >0(i = 1, 2, 3) are intrinsic rates of increase or decrease, $\alpha > 0$ and $\beta > 0$ are parameters representing competitive effects between two prey, η >0 and $\mu > 0$, $\frac{\eta x_1(t)}{1+\omega_1 x_1(t)}$ and $\frac{\mu x_2(t)}{1+\omega_2 x_2(t)}$ are the Holling type II functional responses, $d > 0$ is the rate of conversing prey into predator. $\Delta x_i(t) = x_i(t^+)-x_i(t)$, i = 1, 2, $\Delta z(t) = z(t^+)-z(t)$, T is the period of the impulse for predator in order to eradicate both target pests, protect non-target pest (or harmless insect) from extinction and drive target pest to extinction, or control target pests at acceptably low level to prevent an increasing pest populations from causing an economic loss. $n \in z^+$, $z^+ = \{1, 2, ... g$, $pi > 0$(i = 1, 2) is the proportionality constant which represents the rate of mortality due to the applied pesticide. $q > 0$ is the number of predators released each time. We note that any biological or environmental parameters are naturally subject to fluctuation in time. It is necessary and important to consider models with periodic ecological parameters Thus, the assumption of periodicity of the parameters is a way of incorporating the periodicity of the environment. Furthermore, for simplification, we assume that there is no pulse in system. Based on the point of view, system (1) can be modified as the form:

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left[b_1(t) - x_1(t) - \alpha(t)x_2(t) - \frac{\eta(t)z(t)}{1+\omega_1(t)x_1(t)}\right], \\ \dot{x}_2(t) = x_2(t)\left[b_2(t) - \beta(t)x_1(t) - x_2(t) - \frac{\eta(t)z(t)}{1+\omega_2(t)x_2(t)}\right], \\ \dot{z}(t) = z(t)\left[-b_3(t) + \frac{d(t)\eta(t)x_1(t)}{1+\omega_1(t)x_1(t)} + \frac{d(t)\mu(t)x_2(t)}{1+\omega_2(t)x_2(t)}\right]. \end{cases} \tag{2}$$

Here we give the initial conditions as follows

$$x_i(0) = \varphi_i(0) > 0 \quad (i = 1, 2), \quad z(0) = \varphi_3(0) > 0. \tag{3}$$

Throughout the paper, we always assume that

(H1) For any $t \in R$, $b_i(t)(i = 1, 2, 3)$, $\omega_j(t)(j = 1, 2)$, $\alpha(t)$, $\beta(t)$, $\eta(t)$, $\mu(t)$, $d(t)$ are all non-negative continuous ω periodic functions, i.e., $b_i(t + \omega) = b_i(t)(i = 1, 2, 3)$, $\omega_j(t + \omega) = \omega_j(t)(j = 1, 2)$, $\alpha(t + \omega) = \alpha(t)$, $\beta(t + \omega) = \beta(t)$, $\eta(t + \omega) = \eta(t)$, $\mu(t + \omega) = \mu(t)$, $d(t + \omega) = d(t)$.

The principle object of this article is to find a set of sufficient conditions that guarantee the existence of at least a positive periodic solution and global attractivity of periodic solutions for system (2)-(3). There are some papers which deal with this topic [13,20-25].

The paper is organized as follows: In Section "Basic lemma", we introduce some basic Lemmas. In Section "Existence of positive periodic solutions", sufficient conditions are established for the existence of positive periodic solutions of system (2)-(3). In Section "Uniqueness and global attractivity", by means of suitable Lyapunov functionals, a set of sufficient conditions are derived for the uniqueness and global attractivity of positive periodic solutions of system (2)-(3).

Basic lemma

In order to explore the existence of positive periodic solutions of (2)-(3) and for the reader's convenience, we shall first summarize below a few concepts and results without proof, borrowing from [11].

Let X, Y be normed vector spaces, $L : \mathrm{Dom}L \in X \to Y$ is a linear mapping, $N : X \to Y$ is a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \mathrm{Ker}L = \mathrm{codim}\,\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I - Q)$, it follows that $L|\mathrm{Dom}L \cap \mathrm{Ker}P : (I - P)X \to \mathrm{Im}L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exist isomorphisms $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 1. ([11] Continuation Theorem) *Let L be a Fredholm mapping of index zero and let N be L-compact on* $\bar{\Omega}$. *Suppose*

(a) *for each* $\lambda \in (0, 1)$, *every solution x of* $Lx = \lambda Nx$ *is such that* $x \notin \partial\Omega$;

(b) $QNx \neq 0$ *for each* $x \in \mathrm{Ker}L \cap \partial\Omega$, *and* $\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} \neq 0$, *then the equation* $Lx = Nx$ *has at least one solution lying in* $\mathrm{Dom}L \cap \bar{\Omega}$.

Lemma 2. $R^3_+ = \{((x_1(t), x_2(t), z(t))^T \in R^3 | x_1(t) > 0, x_2(t) > 0, z(t) > 0\}$ *is positive invariant with respect to system* (2)-(3).

Proof. In fact,

$$\begin{cases} \dot{x}_1(t) = \varphi_1(0) \exp\left\{\int_0^t \left[b_1(s) - x_1(s) - \alpha(s)x_2(s) - \frac{\eta(s)z(s)}{1+\omega_1(s)x_1(s)}\right]\mathrm{d}s\right\}, \\ \dot{x}_2(t) = \varphi_2(0) \exp\left\{\int_0^t \left[b_2(s) - \alpha(s)x_1(s) - x_2(s) - \frac{\eta(s)z(s)}{1+\omega_2(s)x_2(s)}\right]\mathrm{d}s\right\}, \\ \dot{z}(t) = \varphi_2(0) \exp\left\{\int_0^t \left[-b_3(s) + \frac{d(s)\eta(s)x_1(s)}{1+\omega_1(s)x_1(s)} + \frac{d(s)\mu(s)x_2(s)}{1+\omega_2(s)x_2(s)}\right]\mathrm{d}s\right\}. \end{cases}$$

Obviously, the conclusion follows.

Existence of positive periodic solutions

For convenience and simplicity in the following discussion, we always use the notations below throughout the paper:

$$\bar{g} = \frac{1}{\omega}\int_0^\omega g(t)\mathrm{d}t, \quad g^L = \min_{t\in[0,\omega]} g(t), \quad g^M = \max_{t\in[0,\omega]} g(t),$$

where $g(t)$ is an ω continuous periodic function. In the following, we will ready to state and prove our result.

Theorem 1. *Let* K_1, K_2, K_4 *and* K_5 *are* defined *by* (19), (23), (32) *and* (36), *respectively. In addition to* (H1), *if the following conditions* (H2) *and* (H3)

$$(H2)\quad \bar{b}_1 > \max\{\exp\{-K_1\} + \bar{\alpha}\exp\{-K_2\}, \exp\{K_1\} + \bar{\alpha}\exp\{K_2\}, \exp\{-K_5\} + \bar{\alpha}\exp\{-K_4\}, \exp\{K_5\} + \bar{\alpha}\exp\{K_4\}\},$$

$$(H3)\quad b_3^M > \max\left\{d^M\eta^M\exp\{K_1\}, d^M\mu^M\exp\{K_4\}\right\}$$

hold, then system (2)-(1.3) *has at least one* ω *periodic solution.*

Proof. Since solutions of (2)-(3) remain positive for all $t \geq 0$, we let

$$u_1(t) = \ln[x_1(t)], \quad u_2(t) = \ln[x_2(t)], \quad u_3(t) = \ln[z(t)]. \tag{4}$$

Substituting (4) into (2), we obtain

$$\begin{cases} \dot{u}_1(t) = b_1(t) - \exp\{u_1(t)\} - \alpha(t)\exp\{u_2(t)\} - \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}, \\ \dot{u}_2(t) = b_2(t) - \beta(t)\exp\{u_1(t)\} - \exp\{u_2(t)\} - \frac{\mu_1(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}, \\ \dot{u}_3(t) = -b_3(t) + \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}. \end{cases} \tag{5}$$

It is easy to see that if system (5) has one ω periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^T$, then $(x_1(t), x_2(t), y(t))^T = (\exp\{u_1(t)\}, \exp\{u_2(t)\}, \exp\{u_3(t)\})^T$ is a positive solution of system (2). Therefore, to complete the proof, it suffices to show that system (5) has at least one ω periodic solution.

Let $X = Z = u(t) = \{(u_1(t), u_2(t), u_3(t))^T \mid u(t) \in C(R, R^3), u(t+\omega) = u(t)\}$, and define $||u|| = ||(u_1(t); u_2(t), u_3(t))^T|| = \max_{t\in[0,\omega]}|u_1(t)| + \max_{t\in[0,\omega]}|u_2(t)| + \max_{t\in[0,\omega]}|u_3(t)|$. Then X and Z are Banach spaces when they are endowed with the norm $||\cdot||$. Let $L : \mathrm{Dom}L \in X \to Z$ and $N : X \to Z$ be the following:

$$Lu = \dot{u}(t),$$

$$Nu = \begin{pmatrix} b_1(t) - \exp\{u_1(t)\} - \alpha(t)\exp\{u_2(t)\} - \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} \\ b_2(t) - \beta(t)\exp\{u_1(t)\} - \exp\{u_2(t)\} - \frac{\mu_1(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}} \\ -b_3(t) + \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}} \end{pmatrix} \tag{6}$$

Define continuous projective operators P and Q:

$$Pu = \frac{1}{\omega}\int_0^\omega u(t)\mathrm{d}t, \; Qu = \frac{1}{\omega}\int_0^\omega u(t)\mathrm{d}t, \; u \in X, \; u \in Z.$$

We can see that $\mathrm{Ker}L = \{u \in X | u = h \in R^3\}$, $\mathrm{Im}L = \{u \in Z | \int_0^\omega u(t)\mathrm{d}t = 0\}$ is closed in X and $\dim(\mathrm{KerL}) = 3 = \mathrm{codim}(\mathrm{ImL})$, then it follows that L is a Fredholm mapping of index zero. Moreover, it is easy to check that

$$QNu = \begin{pmatrix} \frac{1}{\omega}\int_0^\omega \left[b_1(t) - \exp\{u_1(t)\} - \alpha(t)\exp\{u_2(t)\} - \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\right]\mathrm{d}t \\ \frac{1}{\omega}\int_0^\omega \left[b_2(t) - \beta(t)\exp\{u_1(t)\} - \exp\{u_2(t)\} - \frac{\mu_1(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]\mathrm{d}t \\ \frac{1}{\omega}\int_0^\omega \left[-b_3(t) + \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]\mathrm{d}t \end{pmatrix}$$

$$K_P(I-Q)Nu = \begin{pmatrix} \int_0^t \left[b_1(s) - \exp\{u_1(s)\} - \alpha(s)\exp\{u_2(s)\} - \frac{\eta(s)\exp\{u_3(s)\}}{1+\omega_1(s)\exp\{u_1(s)\}}\right] \mathrm{d}s \\ \int_0^t \left[b_2(s) - \beta(s)\exp\{u_1(s)\} - \exp\{u_2(s)\} - \frac{\mu_1(s)\exp\{u_3(s)\}}{1+\omega_2(s)\exp\{u_2(s)\}}\right] \mathrm{d}s \\ \int_0^t \left[-b_3(s) + \frac{d(s)\eta(s)\exp\{u_1(s)\}}{1+\omega_1(s)\exp\{u_1(s)\}} + \frac{d(s)\mu(s)\exp\{u_2(s)\}}{1+\omega_2(s)\exp\{u_2(s)\}}\right]\mathrm{d}s \end{pmatrix}$$
$$- \begin{pmatrix} \frac{1}{\omega}\int_0^\omega \int_0^t \left[b_1(s) - \exp\{u_1(s)\} - \alpha(s)\exp\{u_2(s)\} - \frac{\eta(s)\exp\{u_3(s)\}}{1+\omega_1(s)\exp\{u_1(s)\}}\right] dsdt \\ \frac{1}{\omega}\int_0^\omega \int_0^t \left[b_2(s) - \beta(s)\exp\{u_1(s)\} - \exp\{u_2(s)\} - \frac{\mu_1(s)\exp\{u_3(s)\}}{1+\omega_2(s)\exp\{u_2(s)\}}\right] dsdt \\ \frac{1}{\omega}\int_0^\omega \int_0^t \left[-b_3(s) + \frac{d(s)\eta(s)\exp\{u_1(s)\}}{1+\omega_1(s)\exp\{u_1(s)\}} + \frac{d(s)\mu(s)\exp\{u_2(s)\}}{1+\omega_2(s)\exp\{u_2(s)\}}\right]dsdt \end{pmatrix} \tag{7}$$
$$- \begin{pmatrix} \left(\frac{t}{\omega}-\frac{1}{2}\right)\int_0^\omega \left[b_1(s) - \exp\{u_1(s)\} - \alpha(s)\exp\{u_2(s)\} - \frac{\eta(s)\exp\{u_3(s)\}}{1+\omega_1(s)\exp\{u_1(s)\}}\right] \mathrm{d}s \\ \left(\frac{t}{\omega}-\frac{1}{2}\right)\int_0^\omega \left[b_2(s) - \beta(s)\exp\{u_1(s)\} - \exp\{u_2(s)\} - \frac{\mu_1(s)\exp\{u_3(s)\}}{1+\omega_2(s)\exp\{u_2(s)\}}\right] \mathrm{d}s \\ \left(\frac{t}{\omega}-\frac{1}{2}\right)\int_0^\omega \left[-b_3(s) + \frac{d(s)\eta(s)\exp\{u_1(s)\}}{1+\omega_1(s)\exp\{u_1(s)\}} + \frac{d(s)\mu(s)\exp\{u_2(s)\}}{1+\omega_2(s)\exp\{u_2(s)\}}\right] \mathrm{d}s \end{pmatrix}.$$

Obviously, QN and K_P $(I - Q)N$ are continuous. Since X is a finite-dimensional Banach space, using the Ascoli-Arzela theorem, it is not difficult to show that $\overline{K_P(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is L-compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we are at the point to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lu = \lambda Nu$, $\lambda \in (0, 1)$, we have

$$\begin{cases} \dot{u}_1(t) = \lambda\left[b_1(t) - \exp\{u_1(t)\} - \alpha(t)\exp\{u_2(t)\} - \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\right], \\ \dot{u}_2(t) = \lambda\left[b_2(t) - \beta(t)\exp\{u_1(t)\} - \exp\{u_2(t)\} - \frac{\mu_1(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right], \\ \dot{u}_3(t) = \lambda\left[-b_3(t) + \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]. \end{cases} \tag{8}$$

Suppose that $u(t) = (u_1(t), u_2(t), u_3(t))^T \in X$ is an arbitrary solution of system (8) for a certain $\lambda \in (0, 1)$, integrating both sides of (8) over the interval $[0, \omega]$ with respect to t, we obtain

$$\begin{cases} \int_0^\omega \left[\exp\{u_1(t)\} + \alpha(t)\exp\{u_2(t)\} + \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\right]\mathrm{d}t = \bar{b}_1\omega, \\ \int_0^\omega \left[\beta(t)\exp\{u_1(t)\} + \exp\{u_2(t)\} + \frac{\mu(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]\mathrm{d}t = \bar{b}_2\omega, \\ \int_0^\omega \left[\frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]\mathrm{d}t = \bar{b}_3\omega. \end{cases} \tag{9}$$

In view of (8) and (9), we have

$$\begin{aligned} \int_0^\omega |\dot{u}_1(t)|\mathrm{d}t &= \lambda\int_0^\omega \left|b_1(t) - \exp\{u_1(t)\} - \alpha(t)\exp\{u_2(t)\} - \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\right|\mathrm{d}t \\ &\le \int_0^\omega b_1(t)\mathrm{d}t + \int_0^\omega \left[\exp\{u_1(t)\} + \alpha(t)\exp\{u_2(t)\} + \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\right]\mathrm{d}t \\ &= 2\int_0^\omega b_1(t)\mathrm{d}t = 2\bar{b}_1\omega, \end{aligned} \tag{10}$$

$$\begin{aligned} \int_0^\omega |\dot{u}_2(t)|\mathrm{d}t &= \lambda\int_0^\omega \left|b_2(t) - \beta(t)\exp\{u_1(t)\} - \exp\{u_2(t)\} - \frac{\mu_1(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right|\mathrm{d}t \\ &\le \int_0^\omega b_1(t)\mathrm{d}t + \int_0^\omega \left[\beta(t)\exp\{u_1(t)\} + \exp\{u_2(t)\} + \frac{\mu_1(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]\mathrm{d}t \\ &= 2\int_0^\omega b_2(t)\mathrm{d}t = 2\bar{b}_2\omega, \end{aligned} \tag{11}$$

$$\begin{aligned}\int_0^\omega |\dot{u}_3(t)|\mathrm{d}t &= \lambda \int_0^\omega \left| -b_3(t) + \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}} \right|\mathrm{d}t \\ &\le \int_0^\omega b_3(t)\mathrm{d}t + \int_0^\omega \left[\frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}} + \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\right]\mathrm{d}t \\ &= 2\int_0^\omega b_3(t)\mathrm{d}t = 2\bar{b}_3\omega.\end{aligned} \quad (12)$$

Since $u = (u_1, u_2, u_3)^T \in X$, then there exist ξ_i, $\eta_i \in [0, \omega]$ such that

$$u_i(\xi_i) = \min_{t\in[0,\omega]} u_i(t),\ u_i(\eta_i) = \min_{t\in[0,\omega]} u_i(t),\ i = 1, 2.3.$$

It is easy to see that $u_i(\xi_i) = 0$, $u_i(\eta_i) = 0 (i = 1, 2, 3)$. From this and (8), we have

$$\begin{cases} b_1(\xi_1) - \exp\{u_1(\xi_1)\} - \alpha(\xi_1)\exp\{u_2(\xi_1)\} - \frac{\eta(\xi_1)\exp\{u_3(\xi_1)\}}{1+\omega_1(\xi_1)\exp\{u_1(\xi_1)\}} = 0, \\ b_2(\xi_2) - \beta(\xi_2)\exp\{u_1(\xi_2)\} - \exp\{u_2(\xi_2)\} - \frac{\mu_1(\xi_2)\exp\{u_3(\xi_2)\}}{1+\omega_2(\xi_2)\exp\{u_2(\xi_2)\}} = 0, \\ -b_3(\xi_3) + \frac{d(\xi_3)\eta(\xi_3)\exp\{u_1(\xi_3)\}}{1+\omega_1(\xi_3)\exp\{u_1(\xi_3)\}} + \frac{d(\xi_3)\mu(\xi_3)\exp\{u_2(\xi_3)\}}{1+\omega_2(\xi_3)\exp\{u_2(\xi_3)\}} = 0 \end{cases} \quad (13)$$

and

$$\begin{cases} b_1(\eta_1) - \exp\{u_1(\eta_1)\} - \alpha(\eta_1)\exp\{u_2(\eta_1)\} - \frac{\eta(\eta_1)\exp\{u_3(\eta_1)\}}{1+\omega_1(\eta_1)\exp\{u_1(\eta_1)\}} = 0, \\ b_2(\eta_2) - \beta(\eta_2)\exp\{u_1(\eta_2)\} - \exp\{u_2(\eta_2)\} - \frac{\mu_1(\eta_2)\exp\{u_3(\eta_2)\}}{1+\omega_2(\eta_2)\exp\{u_2(\eta_2)\}} = 0, \\ -b_3(\eta_3) + \frac{d(\eta_3)\eta(\eta_3)\exp\{u_1(\eta_3)\}}{1+\omega_1(\eta_3)\exp\{u_1(\eta_3)\}} + \frac{d(\eta_3)\mu(\eta_3)\exp\{u_2(\eta_3)\}}{1+\omega_2(\eta_3)\exp\{u_2(\eta_3)\}} = 0. \end{cases} \quad (14)$$

It follows from the first and the second equation of (13) that

$$\exp\{u_1(\xi_1)\} < b_1(\xi_1) = b_1^L,\ \exp\{(u_2(\xi_2)\} < b_2(\xi_2) = b_2^L$$

which leads to

$$u_1(\xi_1) < \ln[b_1^L], u_2(\xi_2) < \ln[b_2^L]. \quad (15)$$

In the sequel, we consider two cases.

Case 1. If $u_1(\eta_1) \ge u_2(\eta_2)$, then from the third equation of (14), we get

$$\begin{aligned} b_3^M = b_3(\eta_3) &< d(\eta_3)\eta(\eta_3)\exp\{u_1(\eta_3)\} + d(\eta_3)\eta(\eta_3)\exp\{u_2(\eta_3)\} \\ &\le d^M\eta^M\exp\{u_1(\eta_1)\} + d^M\eta^M\exp\{u_1(\eta_1)\} \\ &= (d^M\eta^M + d^M\mu^M)\exp\{u_1(\eta_1)\}. \end{aligned}$$

Then we have

$$u_1(\eta_1) > \ln\left[\frac{b_3^M}{d^M\eta^M + d^M\mu^M}\right]. \quad (16)$$

By (10), (15) and (16), we can obtain

$$u_1(t) \le u_1(\xi_1) + \int_0^\omega |\dot{u}_1(t)|\mathrm{d}t \le \ln[b_1^L] + 2\bar{b}_1\omega := B_1, \quad (17)$$

$$u_1(t) \geq u_1(\eta_1) - \int_0^{\omega} |\dot{u}_1(t)|\mathrm{d}t \geq \ln\left[\frac{b_3^M}{d^M\eta^M + d^M\mu^M}\right] - 2\bar{b}_1\omega := B_2. \tag{18}$$

It follows from (17) and (18) that

$$\max_{t\in[0,\omega]} |u_1(t)| \leq \max\{|B_1|,\ |B_2|\} := K_1. \tag{19}$$

From the third equation of (14), we derive

$$\begin{aligned} b_3^M = b_3(\eta_3) &< d(\eta_3)\eta(\eta_3)\exp\{u_1(\eta_3)\} + d(\eta_3)\eta(\eta_3)\exp\{u_2(\eta_3)\} \\ &\leq d^M\eta^M \exp\{K_1\} + d^M\eta^M \exp\{u_2(\eta_2)\} \end{aligned}$$

which leads to

$$u_2(\eta_2) > \ln\left[\frac{b_3^M - d^M\eta^M \exp\{K_1\}}{d^M\mu^M}\right]. \tag{20}$$

In view of (11), (15) and (20), we can obtain

$$u_2(t) \leq u_2(\xi_2) + \int_0^{\omega} |\dot{u}_2(t)|\mathrm{d}t \leq \ln[b_2^L] + 2\bar{b}_2\omega := B_3, \tag{21}$$

$$u_2(t) \geq u_2(\eta_2) - \int_0^{\omega} |\dot{u}_2(t)|\mathrm{d}t \geq \ln[\frac{b_3^M - d^M\eta^M \exp\{K_1\}}{d^M\mu^M}] - 2\bar{b}_2\omega := B_4. \tag{22}$$

It follows from (21) and (22) that

$$\max_{t\in[0,\omega]} |u_2(t)| \leq \max\{|B_3|,\ |B_4|\} := K_2. \tag{23}$$

From the first equation of (9), we get

$$\int_0^{\omega} \left[\exp\{-K_1\} + \alpha(t)\exp\{-K_2\} + \frac{\eta(t)\exp\{u_3(\xi_3)\}}{1 + \omega_1^M \exp\{-K_1\}}\right]\mathrm{d}t < \bar{b}_1\omega,$$

$$\int_0^{\omega} \left[\exp\{K_1\} + \alpha(t)\exp\{K_2\} + \eta(t)\exp\{u_3(\eta_3)\}\right]\mathrm{d}t > \bar{b}_1\omega,$$

which reduces to

$$\begin{aligned} &\exp\{-K_1\} + \bar{\alpha}\exp\{-K_2\} + \frac{\bar{\eta}\exp\{u_3(\xi_3)\}}{1 + \omega_1^M \exp\{-K_1\}} < \bar{b}_1, \\ &\exp\{K_1\} + \bar{\alpha}\exp\{K_2\} + \bar{\eta}\exp\{u_3(\eta_3)\} > \bar{b}_1. \end{aligned}$$

Therefore, we have

$$u_3(\xi_3) < \ln\left[\frac{(\bar{b}_1 - \exp\{-K_1\} - \bar{\alpha}\exp\{-K_2\})(1 + \omega_1^M \exp\{-K_1\})}{\bar{\eta}}\right], \tag{24}$$

$$u_3(\eta_3) > \ln\left[\frac{\bar{b}_1 - \exp\{K_1\} - \bar{\alpha}\exp\{K_2\}}{\bar{\eta}}\right]. \tag{25}$$

By (3.9), (24) and (25), we can obtain

$$u_3(t) \le u_3(\xi_3) + \int_0^\omega |\dot{u}_3(t)|\mathrm{d}t \le \ln\left[\frac{(\bar{b}_1 - \exp\{-K_1\} - \bar{\alpha}\exp\{-K_2\})(1 + \omega_1^M \exp\{-K_1\})}{\bar{\eta}}\right] + 2\bar{b}_3\omega := B_5, \tag{26}$$

$$u_3(t) \ge u_3(\eta_3) + \int_0^\omega |\dot{u}_3(t)|\mathrm{d}t \ge \ln\left[\frac{(\bar{b}_1 - \exp\{K_1\} - \bar{\alpha}\exp\{-K_2\})}{\bar{\eta}}\right] - 2\bar{b}_3\omega := B_6. \tag{27}$$

It follows from (26) and (27) that

$$\max_{t\in[0,\omega]} |u_3(t)| \le \max\{|B_5|,\ |B_6|\} := K_3. \tag{28}$$

Case 2. If $u_1(\eta_1) < u_2(\eta_2)$, then from the third equation of (14), we get

$$\begin{aligned} b_3^M = b_3(\eta_3) &< d(\eta_3)\eta(\eta_3)\exp\{u_1(\eta_3)\} + d(\eta_3)\eta(\eta_3)\exp\{u_2(\eta_3)\} \\ &< d^M\eta^M \exp\{u_1(\eta_1)\} + d^M\eta^M \exp\{u_2(\eta_2)\} \\ &= (d^M\eta^M + d^M\mu^M)\exp\{u_2(\eta_2)\}. \end{aligned}$$

Then we have

$$u_2(\eta_2) > \ln\left[\frac{b_3^M}{d^M\eta^M + d^M\mu^M}\right]. \tag{29}$$

By (11), (15) and (29), we can obtain

$$u_2(t) \le u_2(\xi_2) + \int_0^\omega |\dot{u}_2(t)|\mathrm{d}t \le \ln[b_2^L] + 2\bar{b}_2\omega := B_7, \tag{30}$$

$$u_2(t) \ge u_2(\eta_2) - \int_0^\omega |\dot{u}_2(t)|\mathrm{d}t \ge \ln\left[\frac{b_3^M}{d^M\eta^M + d^M\mu^M}\right] - 2\bar{b}_2\omega := B_8. \tag{31}$$

It follows from (30) and (31) that

$$\max_{t\in[0,\omega]} |u_2(t)| \le \max\{|B_7|,\ |B_8|\} := K_4. \tag{32}$$

From the third equation of (14), we derive

$$\begin{aligned} b_3^M = b_3(\eta_3) &< d(\eta_3)\eta(\eta_3)\exp\{u_1(\eta_3)\} + d(\eta_3)\eta(\eta_3)\exp\{u_2(\eta_3)\} \\ &\le d^M\eta^M \exp\{u_1(\eta_1)\} + d^M\eta^M \exp\{K_4\} \end{aligned}$$

which leads to

$$u_1(\eta_1) > \ln\left[\frac{b_3^M - d^M\eta^M\exp\{K_4\}}{d^M\mu^M}\right]. \tag{33}$$

In view of (10), (15) and (33), we can obtain

$$u_1(t) \le u_1(\xi_1) + \int_0^\omega |\dot{u}_1(t)|\mathrm{d}t \le \ln[b_1^L] + 2\bar{b}_1\omega := B_9, \tag{34}$$

$$u_1(t) \geq u_1(\eta_1) - \int_0^{\omega} |\dot{u}_1(t)|\mathrm{d}t \geq \ln\left[\frac{b_3^M - d^M\eta^M \exp\{K_4\}}{d^M\mu^M}\right] - 2\bar{b}_1\omega := B_{10}. \tag{35}$$

It follows from (34) and (35) that

$$\max_{t\in[0,\omega]} |u_1(t)| \leq \max\{|B_9|, |B_{10}|\} := K_5. \tag{36}$$

From the first equation of (9), we get

$$\int_0^{\omega} \left[\exp\{-K_5\} + \alpha(t)\exp\{-K_4\} + \frac{\eta(t)\exp\{u_3(\xi_3)\}}{1+\omega_1^M \exp\{K_5\}}\right]\mathrm{d}t < \bar{b}_1\omega,$$

$$\int_0^{\omega} [\exp\{K_5\} + \alpha(t)\exp\{K_4\} + \eta(t)\exp\{u_3(\eta_3)\}]\mathrm{d}t > \bar{b}_1\omega.$$

Then

$$\exp\{-K_5\} + \bar{\alpha}\exp\{-K_4\} + \frac{\bar{\alpha}\exp\{u_3(\xi_3)\}}{1+\omega_1^M\exp\{K_5\}} < \bar{b}_1,$$

$$\exp\{K_5\} + \bar{\alpha}\exp\{K_4\} + \bar{\eta}\exp\{u_3(\eta_3)\} > \bar{b}_1.$$

Therefore we have

$$u_3(\xi_3) < \ln\left[\frac{(\bar{b}_1 - \exp\{-K_5\} - \bar{\alpha}\exp\{-K_4\})(1+\omega_1^M\exp\{K_5\})}{\bar{\eta}}\right], \tag{37}$$

$$u_3(\eta_3) > \ln\left[\frac{\bar{b}_1 - \exp\{-K_5\} - \bar{\alpha}\exp\{-K_4\}}{\bar{\eta}}\right]. \tag{38}$$

By (3.9), (37) and (38), we can obtain

$$\begin{aligned} u_3(t) &\leq u_3(\xi_3) + \int_0^{\omega} |\dot{u}_3(t)|\mathrm{d}t \leq \ln\left[\frac{(\bar{b}_1 - \exp\{-K_5\} - \bar{\alpha}\exp\{-K_4\})(1+\omega_1^M\exp\{K_5\})}{\bar{\eta}}\right] \\ &\quad + 2\bar{b}_3\omega := B_{11}, \end{aligned} \tag{39}$$

$$u_3(t) \geq u_3(\eta_3) - \int_0^{\omega} |\dot{u}_3(t)|\mathrm{d}t \geq \ln\left[\frac{\bar{b}_1 - \exp\{-K_5\} - \bar{\alpha}\exp\{-K_4\}}{\bar{\eta}}\right] - 2\bar{b}_3\omega := B_{12}. \tag{40}$$

It follows from (39) and (40) that

$$\max_{t\in[0,\omega]} |u_3(t)| \leq \max\{|B_{11}|, |B_{12}|\} := K_6. \tag{41}$$

Obviously, $B_i (i = 1, 2, 3, ..., 12)$ are independent of $\lambda \in (0, 1)$. Take $M = \max\{K_1, K_5\} + \max\{K_2, K4\} + \max\{K_3, K_6\} + K_0$, where K_0 is taken sufficiently large such that every solution $(\tilde{u}_1, \tilde{u}_2, \tilde{u}_3)^T \in R^3$ of the following algebraic equations

$$\begin{cases} \bar{b}_1 - \exp\{u_1\} - \bar{\alpha}\exp\{u_2\} - \dfrac{1}{\omega}\int_0^{\omega} \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\mathrm{d}t = 0, \\ \bar{b}_2 - \bar{\beta}\exp\{u_1\} - \exp\{u_2\} - \dfrac{1}{\omega}\int_0^{\omega} \frac{\mu(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\mathrm{d}t = 0, \\ -\bar{b}_3 + \dfrac{1}{\omega}\int_0^{\omega} \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\mathrm{d}t + \dfrac{1}{\omega}\int_0^{\omega} \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\mathrm{d}t = 0. \end{cases} \tag{42}$$

satisfies $\max_{t\in[0,\omega]}|\tilde{u}_1| + \max_{t\in[0,\omega]}|\tilde{u}_2| + \max_{t\in[0,\omega]}|\tilde{u}_2| < K_0$ (if it exists).

Let $\Omega := \{u = \{u(t)\} \in X : ||u|| < M\}$, then it is easy to see that is an open, bounded set in X and verifies requirement (a) of Lemma 1. When $(u_1(t), u_2(t), u_3(t))^T \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap R^3$, $u = \{(u_1, u_2, u_3)^T\}$ is a constant vector in R^3 with $||u|| = ||(u_1(t), u_2(t), u_3(t))^T|| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_2(t)| + \max_{t\in[0,\omega]} |u_3(t)| = M$. Then we have

$$QNu = \begin{pmatrix} \bar{b}_1 - \exp\{u_1\} - \bar{\alpha}\exp\{u_2\} - \frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\mathrm{d}t \\ \bar{b}_2 - \bar{\beta}\exp\{u_1\} - \exp\{u_2\} - \frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\mathrm{d}t \\ -\bar{b}_3 + \frac{1}{\omega}\int_0^\omega \frac{d(t)\mu(t)\exp\{u_1(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\mathrm{d}t + \frac{1}{\omega}\int_0^\omega \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+\omega_2(t)\exp\{u_2(t)\}}\mathrm{d}t \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (43)$$

Now, the only thing left is to verify that condition (b) in Theorem 2.1 is satisfied. To do this, we define

$\varphi : \mathrm{Dom}X \times [0, 1] \to X$ by

$$\phi(u_1, u_2, u_3, \nu) = \begin{pmatrix} \bar{b}_1 - \frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_3(t)\}}{1+w_1(t)\exp\{u_1(t)\}}\mathrm{d}t \\ \bar{b}_2 - \bar{\beta}\exp\{u_1\} - \exp\{u_2\} \\ -\bar{b}_3 + \frac{1}{\omega}\int_0^\omega \frac{d(t)\eta(t)\exp\{u_1(t)\}}{1+w_1(t)\exp\{u_1(t)\}}\mathrm{d}t \end{pmatrix} + \nu \begin{pmatrix} -\exp\{u_1\} - \overline{\alpha}\exp\{u_2\} \\ -\frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_3(t)\}}{1+w_2(t)\exp\{u_2(t)\}}\mathrm{d}t \\ \frac{1}{\omega}\int_0^\omega \frac{d(t)\mu(t)\exp\{u_2(t)\}}{1+w_2(t)\exp\{u_2(t)\}}\mathrm{d}t \end{pmatrix},$$

where $\nu \in [0, 1]$ is a parameter. Due to the homotopy invariance theorem of topology degree and taking $J = I : \mathrm{Im}Q \to \mathrm{Ker}L$, $(u_1, u_2, u_3)^T \to (u_1, u_2, u_3)^T$, we have

$$\begin{aligned} &\deg\left\{JQN(u_1, u_2, u_3)^T; \Omega \cap \mathrm{Ker}L; 0\right\} \\ &= \deg\left\{QN(u_1, u_2, u_3)^T; \Omega \cap \mathrm{Ker}L; 0\right\} \\ &= sign\{\det \begin{pmatrix} \frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_1+u_3\}}{[1+\omega_1(t)\exp\{u_1(t)\}]^2}\mathrm{d}t & 0 & -\frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\mathrm{d}t \\ -\bar{\beta}\exp\{u_1\} & -\exp\{u_2\} & 0 \\ \frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_1\}}{[1+\omega_1(t)\exp\{u_1(t)\}]^2}\mathrm{d}t & 0 & 0 \end{pmatrix} \\ &= -\mathrm{sign}\left[\frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_3(t)\}}{1+\omega_1(t)\exp\{u_1(t)\}}\mathrm{d}t\right]\left[\frac{1}{\omega}\int_0^\omega \frac{\eta(t)\exp\{u_1\}}{[1+\omega_1(t)\exp\{u_1(t)\}]^2}\mathrm{d}t\right]\exp\{u_2\} = -1 \neq 0. \end{aligned}$$

This proves that condition (b) in Lemma 1 is satisfied. By now, we have proved that verifies all requirements of Lemma 1, then it follows that Lu = Nu has at least one solution $(u_1(t), u_2(t), u_3(t))^T$ in $\mathrm{Dom}L \cap \bar{\Omega}$, that is to say, (5) has at least one ω periodic solution in $\mathrm{Dom}L \cap \bar{\Omega}$. Then we know that $(x_1(t), x_2(t), y(t))^T = (\exp\{u_1(t)\}, \exp\{u_2(t)\}, \exp\{u_3(t)\})^T$ is an ω periodic solution of system (2)-(3) with strictly positive components. Hence the proof.

Uniqueness and global attractivity

We now process to the discussion on the global attractivity of the positive ω-periodic solution $(x_1, x_2, x_3)^T$ in Theorem 1. It is immediate that if $(x_1, x_2, x_3)^T$ is globally attractive, then it is in fact unique.

Lemma 3. *Let* ε *be an arbitrary small positive constant and* $(x_1(t), x_2(t), x_3(t))^T$ *be any positive solution of system* (2)-(3). *If the following condition*

$$(H4)\ b_3 > \frac{d^M\eta^M M_1}{1 + \omega_1^L} + \frac{d^M\mu^M M_2}{1 + \omega_2^L M_2}$$

holds, then exists a positive constant t_0 *such that*

$$0 < x_1 < M_1, 0 < x_2 < M_2, 0 < z < M_3 \quad for\ t > t_0,$$

where

$$M_1 > M_1^* = b_1^M + \varepsilon, M_2 > b_2^M + \varepsilon, M_3 > \varphi_3(0) + \varepsilon.$$

Proof. From the first equation of (2), we obtain

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\left[b_1(t) - x_1(t) - \alpha(t)x_2(t) - \frac{\eta(t)z(t)}{1 + \omega_1(t)x_1(t)}\right] \\ &\leq x_1(t)(b_1(t) - x_1(t)).\end{aligned}$$

Then for arbitrary small positive constant ε, there exists a T_1 >0 such that for $t \geq T_1$, there has

$$x_1(t) \leq b_1^M + \varepsilon. \tag{44}$$

From the second equation of (2), we obtain

$$\begin{aligned}\dot{x}_2(t) &= x_2(t)\left[b_2(t) - \beta(t)x_1(t) - x_2(t) - \frac{\mu(t)z(t)}{1 + \omega_2(t)x_2(t)}\right] \\ &\leq x_2(t)(b_2(t) - x_2(t)).\end{aligned}$$

Then for arbitrary small positive constant ε, there exists a $T_2 > 0$ such that for $t \geq T_2$, there has

$$x_2(t) \leq b_2^M + \varepsilon. \tag{45}$$

Since both functions $\frac{d\eta x_1}{1+\omega_1 x_1}$and $\frac{d\eta x_2}{1+\omega_2 x_2}$ are increasing functions with respect to x_1 and x_2, respectively, from the third equation of (2), we get

$$\begin{aligned}\dot{z}(t) &= z(t)\left[-b_3(t) + \frac{d(t)\eta(t)x_1(t)}{1 + \omega_1(t)x_1(t)} + \frac{d(t)\mu(t)x_2(t)}{1 + \omega_2(t)x_2(t)}\right] \\ &\leq z(t)\left[-b_3^L + \frac{d^M\eta^M M_1}{1 + \omega_1^L M_1} + \frac{d^M\mu^M M_2}{1 + \omega_2 M_2}\right].\end{aligned}$$

Under the assumption (H4), we know that *z(t)* is a decreasing function with respect to *t*, Then for arbitrary small positive constant ε, there exists a $T_3 > 0$ such that for $t \geq T_3$, there has

$$z(t) \leq \varphi_3(0) + \varepsilon. \tag{46}$$

Definition 1. *A positive bounded solution* ($x_1(t)$, $x_2(t)$, $z(t)$) *of system* (2)-(3) *is said to globally attractive, if for any other positive solution* $(x_1^*(t),\ x_2^*(t),\ z^*(t))$*of system* (2)-(3), *we have* $\lim\limits_{t\to+\infty}|x_i(t) - x_i^*(t)| = 0, \lim\limits_{t\to+\infty}|z(t) - z^*(t)| = 0, i = 1, 2.$

Definition 2. [26]*Let f be a nonnegative function defined on* [0, +∞) *such that f is integrable on* [0, +∞) *and is uniformly continuous on* [0, +∞), *then* $\lim_{t\to+\infty} f(t) = 0$.

Theorem 2. *Let* σ_1, σ_2 *and* σ_3 *are defined by* (49), (50) *and* (51), *respectively. In addition to* (H1)-(H4), *if there exist positive constants* $\theta_i (i = 1, 2, 3)$ *and* δ *such that* δ = min{ρ_1, ρ_2, ρ_3} >0, *then system* (2)-(1.3) *has a unique positive ω-periodic solution which is globally attractive.*

Proof. Due to the conclusion in Lemma 3, we need only to show that the attractivity of the positive periodic solution of (2)-(3). Let $x^*(t) = (x_1^*(t), x_2^*(t), z^*(t))^T$ be a positive ω-periodic solution of (2)-(3), and $x(t) = (x_1(t), x_2(t), z(t))^T$ be any positive solution of system (2)-(3). It follows from Lemma 3 that there exist positive constants T and M_i (see Lemma 3) such that for all $t \geq T$,

$$0 < x_1(t) < M_1, 0 < x_2(t) < M_2, 0 < z(t) < M_3.$$

We consider the following Lyapunov functional:

$$V(t) = \sum_{i=1}^{2} \theta_i |\ln x_i(t) - \ln x_i^*(t)| + \theta_3 |\ln z(t) - \ln z^*(t)|. \tag{47}$$

Calculating the upper right derivative $D^+V(t)$ of $V(t)$ along the solution of (2), we have

$$\begin{aligned}
D^+V(t) &= \sum_{i=1}^{2} \theta_i \left[\frac{\dot{x}_i(t)}{x_i(t)} - \frac{\dot{x}_i^*(t)}{x_i^*(t)}\right] \mathrm{sgn}(x_i(t) - x_i^*(t)) + \theta_3 \left[\frac{\dot{z}(t)}{z(t)} - \frac{\dot{z}^*(t)}{z^*(t)}\right] \mathrm{sgn}(z(t) - z^*(t)) \\
&= \theta_1 \mathrm{sgn}(x_1(t) - x_1^*(t)) \left[-(x_1(t) - x_1^*(t)) - \alpha(t)(x_2(t) - x_2^*(t)) - \frac{\eta(t)z(t)}{1 + \omega_1(t)x_1(t)}\right. \\
&\quad \left. + \frac{\eta(t)z^*(t)}{1 + \omega_1(t)x_1^*(t)}\right] + \theta_2 \mathrm{sgn}(x_2(t) - x_2^*(t)) \left[-\beta(t)(x_1(t) - x_1^*(t)) - \frac{\mu(t)z(t)}{1 + \omega_2(t)x_2(t)}\right. \\
&\quad \left. + \frac{\mu(t)z^*(t)}{1 + \omega_2(t)x_2^*(t)} - (x_2(t) - x_2^*(t))\right] + \theta_3(z(t) - z^*(t)) \left[\frac{d(t)\eta(t)x_1(t)}{1 + \omega_1(t)x_1(t)}\right. \\
&\quad \left. - \frac{d(t)\eta(t)x_1^*(t)}{1 + \omega_1(t)x_1^*(t)} + \frac{d(t)\mu(t)x_2(t)}{1 + \omega_2(t)x_2(t)} - \frac{d(t)\mu(t)x_2^*(t)}{1 + \omega_2(t)x_2^*(t)}\right] \\
&\leq -\theta_1|x_1(t) - x_1^*(t)| + \theta_1\alpha^M|x_2(t) - x_2^*(t)| + \theta_1\eta^M\omega_1^M M_1|z(t) - z^*(t)| \\
&\quad + \theta_1\eta^M\omega_1^M M_3|x_1(t) - x_1^*(t)| - \theta_2|x_2(t) - x_2^*(t)| + \theta_2\beta^M|z(t) - z^*(t)| \\
&\quad - \theta_2\mu^M|z(t) - z^*(t)| + \theta_2\mu^M\omega_2^M M_3|x_2(t) - x_2^*(t)| + \theta_2\mu^M\omega_2^M M_2|z(t) - z^*(t)| \\
&\quad + \theta_3[d^M\eta^M|x_1(t) - x_1^*(t)| + d^M\mu^M|x_2(t) - x_2^*(t)|] \\
&\leq -\delta \left[\sum_{i=1}^{2} |x_i(t) - x_i^*(t)| + |z(t) - z^*(t)|\right],
\end{aligned} \tag{48}$$

where

$$\sigma_1 = \theta_1\eta^M\omega_1^M M_3 + \theta_2\beta^M + \theta_2 d^M\eta^M - \theta_1, \tag{49}$$

$$\sigma_2 = \theta_1\alpha^M + \theta_2\mu^M\omega_2^M M_3 + \theta_3 d^M\mu^M - \theta_2, \tag{50}$$

$$\sigma_3 = \theta_1\eta^M\omega_1^M M_1 + \theta_2\eta^M\omega_2^M M_2 + \theta_2\mu^M. \tag{51}$$

An integration of (48) over $[T, t]$, we obtain that

$$\delta \int_T^t \left[\sum_{i=1}^{2} |x_i(s) - x_i^*(s)| + |z(s) - z^*(s)|\right] \mathrm{d}s \leq V(T) - V(t) \text{ for } t \geq T,$$

which implies

$$\int_T^t \left[\sum_{i=1}^{2} |x_i(s) - x_i^*(s)| + |z(s) - z^*(s)|\right] \mathrm{d}s \leq \frac{V(T)}{\delta} < +\infty.$$

Then it follows from Definition 2 that

$$\lim_{t\to+\infty}|x_i(t)-x_i^*(t)| = 0, \lim_{t\to+\infty}|z(t)-z^*(t)| = 0, \ (i=1,2),$$

which implies that the ω-periodic solution of system (2)-(3) is globally attractive. This completes the proof of Theorem 2.

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Authors' contributions
The authors indicated in parentheses made substantial contributions to the following tasks of research: Drafting the manuscript(Y.F.S); Participating in design of the manuscript(Y.F.S, P.L.L.); Writing and revision of the paper(C.J.X, P.L.L.).

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The authors declare that they have no competing interests.

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